

BOUTET DE MONVEL OPERATORS ON SINGULAR MANIFOLDS

OPérateurs de Boutet de Monvel pour de Variétés Singulières

KARSTEN BOHLEN

ABSTRACT. We construct a Boutet de Monvel calculus for general pseudodifferential boundary value problems defined on a broad class of non-compact manifolds, the class of so-called Lie manifolds with boundary. It is known that this class of non-compact manifolds can be used to model many classes of singular manifolds.

RÉSUMÉ. Nous construisons un calcul des type Boutet de Monvel pour des problèmes de valeurs au bord pseudo-différentiels définés sur une large classe de variétés non-compactes, des variétés de Lie à bord. Il est bien connu que cette classe de variétés non-compactes peut être utilisée pour modéliser des nombreuses classes de variétés singulières.

1. INTRODUCTION

The analysis on singular manifolds has a long history, and the subject is to a large degree motivated by the study of partial differential equations (with or without boundary conditions) and by the generalizations of index theory to the singular setting, e.g. Atiyah-Singer type index theorems. One particular approach is based on the observation first made by A. Connes (cf. [7], section II.5) that groupoids are good models for singular spaces. The pseudodifferential calculus on longitudinally smooth groupoids was developed by B. Monthubert, V. Nistor, A. Weinstein and P. Xu; see e.g. [15]. Later a pseudodifferential calculus on a Lie manifold was constructed in [2] via representations of pseudodifferential operators on a Lie groupoid. This representation also yields closedness under composition. It is important for applications in the study of partial differential equations to pose boundary conditions and to construct a parametrix for general boundary value problems. In our case we consider the following data: a Lie manifold (X, \mathcal{V}) with boundary Y which is an embedded, transversal hypersurface $Y \subset X$ and which is a Lie submanifold of X (cf. [2], [1]).

We will describe a general calculus with pseudodifferential boundary conditions on the Lie manifold with boundary (X, Y, \mathcal{V}) . Special cases of our setup have been considered by Schrohe and Schulze, cf. e.g. [16]. Debord and Skandalis study Boutet de Monvel operators using deformation groupoids, [8].

Acknowledgements. For helpful discussions and remarks I thank Magnus Goffeng, Victor Nistor, Julie Rowlett, Elmar Schrohe and Georges Skandalis. I thank the Deutsche Forschungsgemeinschaft (DFG) for their financial support.

2. BOUTET DE MONVEL'S CALCULUS

Boutet de Monvel's calculus (e.g. [6]) was introduced in 1971. For a detailed account we refer the reader to the book [9]. This calculus provides a convenient and general tool to study the classical boundary value problems (BVP's). Let X be a smooth compact manifold with boundary and fix smooth vector bundles $E_i \rightarrow X$, $F_i \rightarrow \partial X$, $i = 1, 2$. Denote by $P \in \Psi_{tr}^m(M)$ a pseudodifferential operator ([9], p. 20 (1.2.4)) with transmission property ([9], p. 23, (1.2.6)) defined on a suitable smooth neighborhood M of X where P_+ means $P = r^+ P e^+$ the truncation. The transmission property (loc. cit.) ensures that P_+ maps functions smooth up to the boundary to functions which are smooth up to the boundary. Additionally, $G: C^\infty(X) \rightarrow C^\infty(X)$ is a singular Green operator ([9], p. 30), $K: C^\infty(\partial X) \rightarrow C^\infty(X)$

2000 *Mathematics Subject Classification.* Primary 58J32; Secondary 58B34.

Key words and phrases. Boutet de Monvel's calculus, groupoids, Lie manifolds.

is a potential operator ([9], p. 29) and $T: C^\infty(X) \rightarrow C^\infty(\partial X)$ is a trace operator ([9], p. 27). We also have a pseudodifferential operator on the boundary $S \in \Psi^m(\partial X)$.

An operator of order $m \leq 0$ and type 0 in Boutet de Monvel's calculus is a matrix

$$A = \begin{pmatrix} P_+ + G & K \\ T & S \end{pmatrix} : \begin{array}{cc} C^\infty(X, E_1) & C^\infty(X, E_2) \\ \oplus & \rightarrow \oplus \\ C^\infty(\partial X, F_1) & C^\infty(\partial X, F_2) \end{array} \in \mathcal{B}^{m,0}(X, \partial X).$$

The calculus of Boutet de Monvel has the following features:

- If the bundles match, i.e. if $E_1 = E_2 = E$, $F_1 = F_2 = F$ the calculus is **closed under composition**.
- If $F_1 = 0$, $G = 0$ and K , S are not present, we obtain a **classical BVP**, e.g. the Dirichlet problem.
- If $F_2 = 0$ and T , S are not present, the calculus **contains inverses of classical BVP's** whenever they exist.

The proof that two Boutet de Monvel operators composed are again of this type is technical, see e.g. [9], chapter 2. Additionally, the symbolic structure of the operators involves more complicated behavior than that of merely pseudodifferential operators.

3. LIE MANIFOLDS WITH BOUNDARY

In this section we will consider the general setup for the analysis on singular and non-compact manifolds.

Example 3.1. On a compact manifold with boundary M we introduce a Riemannian metric which models a singular structure, where the manifold with boundary is viewed as a compactification of a non-compact manifold with cylindrical end. Precisely, the metric is a product metric $g = g_{\partial M} + dt^2$ in a tubular neighborhood of the boundary (or the far end of the cylinder). The cylindrical end is mapped to a tubular neighborhood of the boundary via the Kondratiev transform $r = e^t$ based on [11]. Assume that we are given a tubular neighborhood of the form $[0, \epsilon) \times \partial M$ and let $(r, x') \in [0, \epsilon) \times \partial M$ be local coordinates. The *b-differential operators* take the form for $n = \dim(M)$

$$P = \sum_{|\alpha| \leq m} a_\alpha(r, x') (r \partial_r)^{\alpha_1} \partial_{x'_2}^{\alpha_2} \cdots \partial_{x'_n}^{\alpha_n} = \sum_{|\alpha| \leq m} a_\alpha (r \partial_r)^{\alpha_1} \partial^{\alpha'}. \quad (*)$$

We observe that the vector fields, which are *local generators*, in this example are $\{r \partial_r, \partial_{x_2}, \dots, \partial_{x_n}\}$.

We consider a locally finitely generated module of vector fields \mathcal{V}_b that has local generators as defined in our example. These are the vector fields that are tangent to the boundary ∂M . An operator of order m in the universal enveloping algebra $P \in \text{Diff}_{\mathcal{V}_b}^m(M)$ is locally written as in (*). Since \mathcal{V}_b is a locally finitely generated and projective $C^\infty(M)$ -module we obtain a vector bundle $\mathcal{A}_b \rightarrow M$ such that the smooth sections identify $\Gamma(\mathcal{A}_b) \cong \mathcal{V}_b$ by the Serre-Swan theorem. On \mathcal{A}_b we have a structure of a *Lie algebroid* with anchor $\varrho: \mathcal{A}_b \rightarrow TM$ (see [2] for further details).

There are sub Lie algebras of \mathcal{V}_b constituting so-called *Lie structures* which model different types of singular structures on a manifold, see also [1], [2], [4]. In this general setup M is a compact manifold *with corners* (generalizing Example 3.1) viewed as the compactification endowed with a Riemannian metric. The Riemannian metric is of product type in a tubular neighborhood of the singular hyperfaces. The topological structure of M is such that M has a finite number of embedded (intersecting) codimension one hypersurfaces. Open subsets of $[-1, 1]^k \times \mathbb{R}^{n-k}$, where k is the codimension, are needed to model manifolds with corners. We can require the transition maps to be smooth and obtain a smooth structure on M . In our setup we consider such a Lie manifold X with an additional hypersurface (denoted Y below) which is *transversal*. Hence Y is allowed to intersect the singular strata (at infinity) of X as long as this intersection does not occur in a corner (where two singular strata meet).

Definition 3.2 ([2], Def. 1.1). A *Lie manifold* (X, \mathcal{A}^\pm) consists of the following data.

- A compact manifold with corners X .
- A Lie algebroid $(\mathcal{A}^\pm, \varrho_\pm)$ with projection map $\pi_\pm: \mathcal{A}^\pm \rightarrow X$.

iii) The module of vector fields $\mathcal{V}_\pm = \Gamma(\mathcal{A}^\pm)$ is a locally finitely generated, projective $C^\infty(X)$ -module.

Definition 3.3 ([1], Def. 2.1, Def. 2.5). A *Lie manifold with boundary* (X, Y, \mathcal{A}^\pm) consists of the following data.

- i) A Lie manifold (X, \mathcal{A}^\pm) .
- ii) An embedded codimension one submanifold with corners $Y \hookrightarrow X$.
- iii) There is a Lie algebroid $(\mathcal{A}_\partial, \varrho_\partial)$ on Y with projection map $\pi_\partial: \mathcal{A}_\partial \rightarrow Y$ such that \mathcal{A}_∂ is a Lie subalgebroid of \mathcal{A}^\pm , [14], Def. 4.3.14.
- iv) The submanifold Y is *transversal*, i.e. $\varrho_\pm(\mathcal{A}_y) + T_y Y = T_y X$, $y \in \partial Y$.
- v) The interior (X_0, Y_0) is diffeomorphic to a smooth manifold with boundary.

Remark 3.4. i) In [1] the authors define a Lie manifold with boundary (X, Y, \mathcal{V}_\pm) and also the *double* of a given Lie manifold with boundary. We denote this double by $M = 2X$ which is a Lie manifold (M, \mathcal{V}) . The *Lie structure* \mathcal{V} is defined such that $\mathcal{V}_\pm = \{V|_{X_\pm} : V \in \mathcal{V}\}$. We obtain a Lie manifold (M, \mathcal{A}) with the Lie algebroid (\mathcal{A}, ϱ) .

ii) We set $\mathcal{W} = \Gamma(\mathcal{A}_\partial)$ for the *Lie structure* of Y . Using iii) and iv) of the definition we obtain (cf. [14], pp. 164-165)

$$\mathcal{W} = \{V \in \Gamma(Y, \mathcal{A}|_Y) : \varrho \circ V \in \Gamma(Y, TY)\} = \{V|_Y : V \in \mathcal{V}, V|_Y \text{ tangent to } Y\}.$$

4. QUANTIZATION

In this section we describe the quantization of Hörmander symbols defined on the conormal bundles. We restrict ourselves to the case of trace operators. The other cases are defined analogously.

We fix the following data:

- A Lie manifold with boundary (X, Y, \mathcal{V}) and the double $M = 2X$ of X , endowed with Lie structure $2\mathcal{V}$. Fix a Lie algebroid $(\pi: \mathcal{A} \rightarrow M, \varrho_M)$ such that $\Gamma(\mathcal{A}) = 2\mathcal{V}$. The hypersurface Y is endowed with the Lie structure \mathcal{W} as defined in 3.4. Furthermore, fix the vector bundle $(\pi_\partial: \mathcal{A}_\partial \rightarrow Y, \varrho_\partial)$ with $\Gamma(\mathcal{A}_\partial) = \mathcal{W}$.
- We fix the *normal bundles* $\mathcal{A}|_Y/\mathcal{A}_\partial =: \mathcal{N} \rightarrow Y$ as well as¹ $\mathcal{N}^\mathcal{X} \Delta_Y \rightarrow Y$, $\mathcal{N}^{\mathcal{X}^t} \Delta_Y \rightarrow Y$, $\mathcal{N}^\mathcal{G} \Delta_Y \rightarrow Y$ which are used to quantize pseudodifferential, trace, potential and singular Green operators respectively.

The notation is reminiscent of the underlying geometry which is described using groupoids and groupoid correspondences [5]. We will keep using this notation, though we remark that there are (non-canonical) isomorphisms

$$\mathcal{N}^\mathcal{X} \Delta_Y \cong \mathcal{A}_\partial \times \mathcal{N}, \quad \mathcal{N}^{\mathcal{X}^t} \Delta_Y \cong \mathcal{N} \times \mathcal{A}_\partial \quad \text{and} \quad \mathcal{N}^\mathcal{G} \Delta_Y \cong \mathcal{A}|_Y \times \mathcal{N}.$$

Remark 4.1. i) On the singular normal bundles we define the Hörmander symbols spaces $S^m(\mathcal{N}^\mathcal{X} \Delta_Y^*) \subset C^\infty(\mathcal{N}^\mathcal{X} \Delta_Y^*)$ as in [10], Thm 18.2.11.

ii) Define the *inverse fiberwise Fourier transform*

$$\mathcal{F}_f^{-1}(\varphi)(\zeta) = \int_{\overline{\pi}(\zeta) = \pi(\xi)} e^{i\langle \xi, \zeta \rangle} \varphi(\xi) d\xi, \quad \varphi \in S(\mathcal{N}^\mathcal{X} \Delta_Y^*).$$

Here we use the notation $S(\mathcal{N}^\mathcal{X} \Delta_Y^*)$ for the space of rapidly decreasing functions on the conormal bundle, see also [17], Chapter 1.5.

The spaces of conormal distributions are defined as $I^m(\mathcal{N}^\mathcal{X} \Delta_Y, \Delta_Y) := \mathcal{F}_f^{-1} S^m(\mathcal{N}^\mathcal{X} \Delta_Y^*)$ and $I^m(\mathcal{N}^{\mathcal{X}^t} \Delta_Y, Y)$, $I^m(\mathcal{N}^\mathcal{G} \Delta_Y, Y)$ analogously.

On a Lie manifold the injectivity radius is positive, see [3], Thm. 4.14. Let r be smaller than the injectivity radius and write $(\mathcal{N}^\mathcal{X} \Delta_Y)_r = \{v \in \mathcal{N}^\mathcal{X} \Delta_Y : \|v\| < r\}$ as well as $I_{(r)}^m(\mathcal{N}^\mathcal{X} \Delta_Y, \Delta_Y) = I^m((\mathcal{N}^\mathcal{X} \Delta_Y)_r, \Delta_Y)$.

Fix the restriction $\mathcal{R}: I_{(r)}^m(\mathcal{N}^\mathcal{X} \Delta_Y, \Delta_Y) \rightarrow I_{(r)}^m(N^{Y_0 \times M_0} \Delta_{Y_0}, \Delta_{Y_0})$. Additionally, denote by \mathcal{J}_{tr} the action of a conormal distribution (its induced linear operator). We denote by Ψ the normal fibration of the inclusion $\Delta_{Y_0} \hookrightarrow Y_0 \times M_0$ such that Ψ is the local diffeomorphism mapping an open neighborhood of the zero section $O_{Y_0} \subset V \subset N^{Y_0 \times M_0} \Delta_{Y_0}$

¹We denote by Δ_Y the diagonal in $Y \times Y$ being understood as a submanifold of $Y \times M$, $M \times Y$ and $M \times M$, groupoids \mathcal{G} , \mathcal{G}_∂ and spaces \mathcal{X} , \mathcal{X}^t as defined in [5].

onto an open neighborhood $\Delta_{Y_0} \subset U \subset Y_0 \times M_0$ (cf. [17], Thm. 4.1.1). Then we have the induced map on conormal distributions $\Psi_*: I_{(r)}^m(N^{Y_0 \times M_0} \Delta_{Y_0}, \Delta_{Y_0}) \rightarrow I^m(Y_0 \times M_0, \Delta_{Y_0})$. Also let $\chi \in C_c^\infty(\mathcal{N}^{\mathcal{X}} \Delta_Y)$ be a cutoff function which acts by multiplication $I^m(\mathcal{N}^{\mathcal{X}} \Delta_Y, \Delta_Y) \rightarrow I_{(r)}^m(\mathcal{N}^{\mathcal{X}} \Delta_Y, \Delta_Y)$.

Definition 4.2 (Quantization). Define $q_{T,\chi}: S^m(\mathcal{N}^{\mathcal{X}} \Delta_Y^*) \rightarrow \mathcal{T}^{m,0}(M, Y)$ such that for $t \in S^m(\mathcal{N}^{\mathcal{X}} \Delta_Y^*)$ we have $q_{T,\chi}(t) = \mathcal{J}_{tr} \circ q_{\Psi,\chi}(t)$ where $q_{\Psi,\chi}(t) = \Psi_*(\mathcal{R}(\chi \mathcal{F}_f^{-1}(t)))$.

From the compactness of M we can associate to each vector field in $2\mathcal{V}$ a *global flow* $2\mathcal{V} \ni V \mapsto \Phi_V: \mathbb{R} \times M \rightarrow M$. Then consider the diffeomorphism $\Phi(1, -): M \rightarrow M$ evaluated at time $t = 1$ and fix the corresponding group actions on functions which we denote by $2\mathcal{V} \ni V \mapsto \varphi_V: C^\infty(M) \rightarrow C^\infty(M)$.

Definition 4.3. The class of \mathcal{V} -trace operators is defined as $\mathcal{T}_{2\mathcal{V}}^{m,0}(M, Y) := \mathcal{T}^{m,0}(M, Y) + \mathcal{T}_{2\mathcal{V}}^{-\infty,0}(M, Y)$. Here $\mathcal{T}^{m,0}(M, Y)$ consists of the extended operators from the previous definition. The residual class is defined as follows

$$\mathcal{T}_{2\mathcal{V}}^{-\infty,0}(M, Y) := \text{span}\{q_{\chi,T}(t)\varphi_{V_1} \cdots \varphi_{V_k}: V_j \in 2\mathcal{V}, \chi \in C_c^\infty(\mathcal{N}^{\mathcal{X}} \Delta_Y), t \in S^{-\infty}(\mathcal{N}^{\mathcal{X}} \Delta_Y^*)\}.$$

We henceforth denote by $\mathcal{B}_{2\mathcal{V}}^{m,0}(M, Y)$ the class of extended Boutet de Monvel operators which consist of matrices of operators $\begin{pmatrix} P+G & K \\ T & S \end{pmatrix}$. The components are given via the fibrations on the appropriate normal bundles.

5. COMPOSITIONS AND PARAMETRICES

To prove closedness under composition we require to assume that a groupoid \mathcal{G} which integrates the Lie structure on M and a groupoid \mathcal{G}_∂ which integrates the Lie structure on Y are chosen in the following sense. Precisely, we construct for the given Lie structures on M and Y respectively integrating groupoids \mathcal{G} and \mathcal{G}_∂ as well as a morphism from $\mathcal{G} \rightarrow \mathcal{G}_\partial$ and a morphism from $\mathcal{G}_\partial \rightarrow \mathcal{G}$ in the category of Lie groupoids. These morphisms are described using *correspondences* of groupoids, see [13].

Example 5.1. Consider the example of the algebroids $\mathcal{A} = TM$, $\mathcal{A}_\partial = TY$, i.e. the Lie structures consisting of *all vector fields*. The pair groupoids $M \times M \rightrightarrows M$, $Y \times Y \rightrightarrows Y$ and also the *path groupoids* (see [12], example 2.9) $\mathcal{P}_M \rightrightarrows M$, $\mathcal{P}_Y \rightrightarrows Y$ integrate these algebroids.

For several Lie structures, groupoids and correspondences with good geometry exist, e.g. the Lie structure of *b*-vector fields or the structure of fibered cusp vector fields [5]. The following results hold for Lie structures of this type.

Theorem 5.2. *The class of extended Boutet de Monvel operators $\mathcal{B}_{2\mathcal{V}}^{0,0}(M, Y)$ is closed under composition and adjoint, hence $\mathcal{B}_{2\mathcal{V}}^{0,0}(M, Y)$ forms an associative $*$ -algebra.*

We define the class of *truncated* Boutet de Monvel operators as follows. The restriction r^+ to the interior $\mathring{X}_0 := X_0 \setminus Y_0$ and the extension by zero operator e^+ are given on the manifold level by

$$L^2(M_0) \begin{matrix} \xleftarrow{r^+} \\ \xrightarrow{e^+} \end{matrix} L^2(\mathring{X}_0)$$

with $r^+e^+ = \text{id}_{L^2(\mathring{X}_0)}$ and e^+r^+ being a projection onto a subspace of $L^2(M_0)$. We define

$$\text{End} \begin{pmatrix} C_c^\infty(M_0) \\ \oplus \\ C_c^\infty(Y_0) \end{pmatrix} \supset \mathcal{B}_{2\mathcal{V}}^{m,0}(M, Y) \ni A = \begin{pmatrix} P+G & K \\ T & S \end{pmatrix} \mapsto \mathcal{C}(A) = \begin{pmatrix} r^+(P+G)e^+ & r^+K \\ Te^+ & S \end{pmatrix} \in \text{End} \begin{pmatrix} C_c^\infty(X_0) \\ \oplus \\ C_c^\infty(Y_0) \end{pmatrix}.$$

Definition 5.3. The class of *truncated operators* is for $m \leq 0$ defined as $\mathcal{B}_{\mathcal{V}}^{m,0}(X, Y) := \mathcal{C} \circ \mathcal{B}_{2\mathcal{V}}^{m,0}(M, Y)$.

To show closedness under composition we use the longitudinally smooth structure of the integrating groupoids as well as the previous Theorem. This enables us to state the second main result.

Theorem 5.4. *The calculus $\mathcal{B}_V^{0,0}(X, Y)$ is closed under composition and adjoint.*

A priori, the inverse of an invertible Boutet de Monvel operator will not be contained in our calculus due to the definition via compactly supported distributional kernels. We define a completion $\overline{\mathcal{B}}_V^{\infty,0}(X, Y)$ of the residual Boutet

de Monvel operators with regard to the family of norms of operators $\mathcal{L} \begin{pmatrix} H_V^t(X) & H_V^r(X) \\ \oplus & \oplus \\ H_W^t(Y) & H_W^r(Y) \end{pmatrix}$ on Sobolev spaces, cf.

[4]. Define the completed algebra of Boutet de Monvel operators as

$$\overline{\mathcal{B}}_V^{0,0}(X, Y) = \mathcal{B}_V^{0,0}(X, Y) + \overline{\mathcal{B}}_V^{\infty,0}(X, Y).$$

The resulting algebra contains inverses and has favorable algebraic properties, e.g. it is spectrally invariant, [5]. We obtain a parametrix construction after defining a notion of *Shapiro-Lopatinski ellipticity*. The indicial symbol \mathcal{R}_F of an operator A on X is an operator $\mathcal{R}_F(A)$ defined as the restriction to a singular hyperface $F \subset X$ (see [2]). Note that if F intersects the boundary Y non-trivially we obtain in this way a non-trivial Boutet de Monvel operator $\mathcal{R}_F(A)$ defined on the Lie manifold F with boundary $F \cap Y$.

Definition 5.5. i) We say that $A \in \overline{\mathcal{B}}_V^{0,0}(X, Y)$ is \mathcal{V} -elliptic if the principal symbol $\sigma(A)$ and the principal boundary symbol $\sigma_\partial(A)$ are both pointwise invertible.

ii) A \mathcal{V} -elliptic operator A is *elliptic* if $\mathcal{R}_F(A)$ is pointwise invertible for each hyperface $F \subset X$.

Theorem 5.6. i) Let $A \in \overline{\mathcal{B}}_V^{0,0}(X, Y)$ be \mathcal{V} -elliptic. There is a parametrix $B \in \overline{\mathcal{B}}_V^{0,0}(X, Y)$ of A , in the sense

$$I - AB \in \overline{\mathcal{B}}_V^{\infty,0}(X, Y), \quad I - BA \in \overline{\mathcal{B}}_V^{\infty,0}(X, Y).$$

ii) Let $A \in \overline{\mathcal{B}}_V^{0,0}(X, Y)$ be elliptic. There is a parametrix $B \in \overline{\mathcal{B}}_V^{0,0}(X, Y)$ of A up to compact operators

$$I - AB \in \mathcal{K} \begin{pmatrix} L_V^2(X) \\ \oplus \\ L_W^2(Y) \end{pmatrix}, \quad I - BA \in \mathcal{K} \begin{pmatrix} L_V^2(X) \\ \oplus \\ L_W^2(Y) \end{pmatrix}.$$

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¹ LEIBNIZ UNIVERSITY HANNOVER, GERMANY

E-mail address: bohlen.karsten@math.uni-hannover.de